Week 2
MATH 4A
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2-2.3 Given the augmented matrix below, solve the associated system of equations. For your variables, use $x_1, x_2, x_3, \cdots, x_8$.

$$\left[\begin{array}{cccccccc|c} 1 & 2 & -2 & -3 & 0 & 8 & -4 & -6 & 9 \\ 0 & 0 & 0 & 0 & 1 & 9 & -7 & 7 & -3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 7 & 7 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & -2 \end{array}\right]$$

First, get this in RREF...

$$\left[\begin{array}{cccccccc|c} 1 & 2 & -2 & -3 & 0 & 0 & 0 & 178 & -151 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 244 & -188 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -21 & 19 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & -2 \end{array}\right]$$

This means: $x_1 = -2x_2 + 2x_3 + 3x_4 - 178x_8 - 151$
$x_5 = -244x_8 - 188$
$x_6 = 21x_8 + 19$
$x_7 = -4x_8 - 2$

So, Solution looks like

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} -2x_2 + 2x_3 + 3x_4 - 178x_8 - 151 \\ x_2 \\ x_3 \\ x_4 \\ -244x_8 - 188 \\ 21x_8 + 19 \\ -4x_8 - 2 \\ x_8 \end{bmatrix}$$

2-2.4 Solve the following system:

$$\begin{cases} x_1 - 4x_2 - 2x_3 \qquad -3x_5 + 4x_6 = -3 \\ \qquad\qquad -x_4 + 3x_5 - 2x_6 = 2 \\ x_1 - 4x_2 \qquad\qquad +7x_5 - 8x_6 = -5 \end{cases}$$

Get Aug. matrix:
$$\left[\begin{array}{cccccc|c} 1 & -4 & -2 & 0 & -3 & 4 & -3 \\ 0 & 0 & 0 & -1 & 3 & -2 & 2 \\ 1 & -4 & 0 & 0 & 7 & -8 & -5 \end{array}\right]$$

RREF:
$$\left[\begin{array}{cccccc|c} 1 & -4 & 0 & 0 & 7 & -8 & -5 \\ 0 & 0 & 1 & 0 & 5 & -6 & -1 \\ 0 & 0 & 0 & 1 & -3 & 2 & -2 \end{array}\right]$$

$\Rightarrow$ $x_1 = 4x_2 - 7x_5 + 8x_6 - 5$

$x_3 = -5x_5 + 6x_6 - 1$

$x_4 = 3x_5 - 2x_6 - 2$

$\Rightarrow$ Solution looks like
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 4x_2 - 7x_5 + 8x_6 - 5 \\ x_2 \\ -5x_5 + 6x_6 - 1 \\ 3x_5 - 2x_6 - 2 \\ x_5 \\ x_6 \end{bmatrix}$$

$\Rightarrow$ So, any solution can be written
$$\begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -7 \\ 0 \\ -5 \\ 3 \\ 1 \\ 0 \end{bmatrix} x_5 + \begin{bmatrix} 8 \\ 0 \\ 6 \\ -2 \\ 0 \\ 1 \end{bmatrix} x_6 + \begin{bmatrix} -5 \\ 0 \\ -1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$$

where x_2, x_5, x_6 are free.

2-2.8 Let $\mathbf{u} = \begin{bmatrix} 9 \\ 3 \\ 4 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 7 \\ 1 \\ -4 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} -9 \\ -4 \\ 8 \end{bmatrix}$.

Compute $8u + 6v - 7w$.

$$8u + 6v - 7w$$

$$= 8\begin{bmatrix} 9 \\ 3 \\ 4 \end{bmatrix} + 6\begin{bmatrix} 7 \\ 1 \\ -4 \end{bmatrix} - 7\begin{bmatrix} -9 \\ -4 \\ 8 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} 72 \\ 24 \\ 32 \end{bmatrix}} \begin{bmatrix} 72 \\ 24 \\ 32 \end{bmatrix} + \begin{bmatrix} 42 \\ 6 \\ -24 \end{bmatrix} + \begin{bmatrix} 63 \\ 28 \\ -56 \end{bmatrix}$$

$$= \begin{bmatrix} 177 \\ 58 \\ -48 \end{bmatrix}$$

2-2.10 Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 4 \\ -5 & 4 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} -2 \\ 4 \\ -14 \end{bmatrix}$.

Determine if b is a linear combination of a_1, a_2, a_3, the columns of A. If so, determine a nontrivial linear combination.

We want to determine if there are c_1, c_2, c_3 such that $\cancel{\begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}}$

$$c_1 \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ -2 \\ 4 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ -14 \end{bmatrix}$$

This is equivalent to $\begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 4 \\ -5 & 4 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ -14 \end{bmatrix}$

which amounts to the following aug. matrix:

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & -2 \\ 0 & -2 & 4 & 4 \\ -5 & 4 & 2 & -14 \end{array}\right]$$

RREF $\Rightarrow$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

Last row tells us this is inconsistent!

So no such c_1, c_2, c_3!!